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Unstable ion sound in plasmas with drifting electrons

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Abstract. The properties of a weakly ionized plasma with a high concentration of flowing neutrals are discussed as well as the excitation of ion sound oscillations propagating obliquely to the magnetic field lines. The model used in the present study includes collisions of plasma species with neutrals in the limit when electrons are magnetized but ions are not. The electron collision frequency is higher than the shifted wave frequency, which allows for the fluid description of the electrons. The ion-neutral collision frequency remains arbitrary and, therefore, the ion species is described by a collisional Boltzman kinetic equation. The electron drift in the perpendicular direction results in the instability of accidentally excited ion sound oscillations, which turn out to be highly unstable for practically all physically acceptable values of the electron drift. In addition, the presence of a population of hotter electrons is shown to reduce the perpendicular electron drift and to increase the instability threshold.

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1 Introduction

Neutrals in a plasma imply a higher degree of complexity and a plethora of processes that are absent in a purely ionized plasma. Electron/proton collisions with neutrals may become dominant, compared to the collisions between charged particles, if the ionization is very weak, such as in the case of the lower layers in the terrestrial ionosphere, in the solar photosphere, and in most of the astrophysical clouds. In certain situations, this may result in a permanent creation and loss of plasma species [1,2], so that an equilibrium state is achieved as a balance between these two processes. Yet, even if such inelastic collisions are absent or negligible, elastic collisions with neutrals may result in novel phenomena and wave instabilities. Under laboratory conditions this may result in the formation of coherent structures like spirals [3] and tripoles [4]. This domain of predominant elastic collisions with neutrals will be discussed also in the present study, in which an ion sound wave is excited by a neutral flow and by the consequent electron drift in the direction perpendicular to both the neutral flow and the magnetic field lines. Such a neutral motion appears naturally in various plasmas. For example, in the case of the very weakly ionized terrestrial ionosphere this comes due to tidal effects [5], while in the case of the Sun the complete photosphere consists of a granular network of convective motion caused by the heating from the solar interior.

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Normally, an ion sound wave in a hot ion plasma is Landau damped, and waves with wavelengths far exceeding the electron Debye radius may propagate if

$$f(\tau) \equiv (z_i\tau)^{3/2}\exp(-z_i\tau/2) \ll 1.$$

Here, $\tau = T_e/T_i$ and z_i is the ion charge number. For an electron-proton plasma $f(\tau)$ becomes completely negligible for $\tau \geq 15$. However, for higher ion charges, e.g. $z_i = 5$, $f(\tau)$ becomes negligible already at $\tau \geq 3$. Thus, highly charged and hot ions practically do not contribute to the Landau damping as long as $\tau \geq 3$, a condition that may be taken as always satisfied.

In the presence of an electron stream described by a velocity v_{e0}, the ion sound in an ordinary electron-proton plasma may become unstable [6]. The increment/decrement is then given by

$$\omega_{im} = \left(\frac{\pi}{8}\right)^{1/2} kc_s \left[\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{v_{e0}}{c_s} - 1\right) - \tau^{3/2}\exp\left(-\frac{\tau}{2}\right)\right]$$

and the mode is unstable provided that

$$v_{e0} > c_s\left[1 + (m_i/m_e)^{1/2} f(\tau)\right]. \quad (1)$$

Here $c_s = (\kappa T_e/m_i)^{1/2}$ is the ion sound speed. For $f(1) = 0.6$ it is seen that the instability develops if $v_{e0} > 27c_s$, while for $f(10) = 0.2$ the threshold is still high, viz. $v_{e0} > 10c_s$.

The physical picture and the stability/instability conditions may become different in a weakly ionized plasma with dominant collisions with neutrals and in the presence of the free energy stored in the flow of the neutral fluid, and these phenomena will be discussed further in the text.

2 Equilibrium with neutral flow

According to reference [7] and many subsequent papers (e.g. [8,9]), in a weakly ionized plasma a macroscopic motion of neutrals perpendicular to the magnetic field structures, like spot regions in the solar photosphere, will drag along the tiny ion population, while electrons have the tendency of staying behind as being stuck to the magnetic field lines, producing in that way a large scale electric field. This is due to the fact that, in spite of the collisions, the electrons may behave as magnetized particles, while ions (protons), in view of their considerably larger mass and due to their collisions with neutrals, are essentially un-magnetized. In the next stage, this induced electric field causes an electron motion in the same direction with an additional retardation of the ion motion, until the moment when a quasi-static state is formed in which neutrals move forward, dragging ions and subsequently electrons in the same direction, all in the presence of the induced electric field. If the neutral motion is in the radial direction (i.e., locally along the x-axis), perpendicular to the magnetic field lines (which may be assumed in the z-direction), this electric field will cause an additional poloidal electron drift (locally in the y-direction). Assuming a quasi-static equilibrium state in which, similar to reference [7], the dominant forces are the electromagnetic and collisional, eventual pressure terms associated with the equilibrium density/temperature gradients are omitted. Note however that the pressure terms are kept in the description of the electron perturbations. This situation is well described by the following set of fluid equations:

$$en_0(\mathbf{E}_0 + \mathbf{v}_{i0} \times \mathbf{B}_0) - m_i n_0 \nu_{in}(\mathbf{v}_{i0} - \mathbf{v}_{n0}) = 0, \quad (2)$$

$$-en_0(\mathbf{E}_0 + \mathbf{v}_{e0} \times \mathbf{B}_0) - m_e n_0 \nu_{en}(\mathbf{v}_{e0} - \mathbf{v}_{n0}) = 0. \quad (3)$$

Here, the standard notation is used, quasi-neutrality is assumed and the collision frequencies include electron and ion collisions with neutrals. In the regime of a quasi steady state and for an ambipolar drift, this yields the electric field in terms of the neutral speed:

$$E_0 = \left[\nu_{en}\left(\nu_{in} + \frac{\Omega_i^2}{\nu_{in}}\right) - \nu_{in}\left(\nu_{en} + \frac{\Omega_e^2}{\nu_{en}}\right)\right] \times \left[\frac{1}{m_i}\left(\nu_{en} + \frac{\Omega_e^2}{\nu_{en}}\right) + \frac{1}{m_e}\left(\nu_{in} + \frac{\Omega_i^2}{\nu_{in}}\right)\right]^{-1} \frac{v_{n0}}{e}. \quad (4)$$

In the case of magnetized electrons and un-magnetized ions, i.e.

$$\frac{\Omega_e}{\nu_{en}} \gg 1, \quad \frac{\Omega_i}{\nu_{in}} \ll 1, \quad (5)$$

the electric field becomes

$$E_0 = -[v_{n0}\nu_{in}\Omega_e^2/(e\nu_{en})][\Omega_e^2/(m_i\nu_{en}) + \nu_{in}/m_e]^{-1}. \quad (6)$$

This can be further simplified if

$$\frac{\Omega_e}{\nu_{en}}\frac{\Omega_i}{\nu_{in}} > 1, \quad (7)$$

yielding [7]

$$E_0 = -\frac{m_i\nu_{in}}{e}v_{n0}. \quad (8)$$

Using conditions (5) and (7), we have $v_{e0x} \sim v_{i0x} \approx v_{n0}\nu_{en}\nu_{in}/(\Omega_e\Omega_i)$ and the electron drift velocity to the leading order terms becomes

$$v_{e0} \equiv v_{e0y} = \frac{\nu_{in}}{\Omega_i}v_{n0}. \quad (9)$$

In view of (5) we note that the magnitude of the electron drift considerably exceeds the neutral flow speed, while the ion drift is negligible.

3 Electron drift driven ion sound

When the conditions given above are satisfied, an ion sound wave may propagate obliquely to the magnetic field lines in such a way that the electron dynamics remain mainly parallel to the magnetic field lines [10]. This is seen from the following. For perturbations $\sim\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, and for frequencies satisfying $\nu_{en} > \omega - \mathbf{k}\cdot\mathbf{v}_{e0}$, we may use the fluid equations for electrons. From the electron perpendicular momentum equation we obtain the total electron perpendicular velocity:

$$\mathbf{v}_{\perp e} = \frac{\Omega_e^2}{\Omega_e^2 + \nu_{en}^2}\left(\frac{1}{B_0}\boldsymbol{e}_z \times \nabla_\perp\phi - \frac{v_{Te}^2}{\Omega_e}\boldsymbol{e}_z \times \frac{\nabla_\perp n}{n} - \frac{v_{Te}^2}{\Omega_e}\frac{\nu_{en}}{\Omega_e}\frac{\nabla_\perp n}{n} + \frac{\nu_{en}}{\Omega_e}\frac{\nabla_\perp\phi}{B_0}\right). \quad (10)$$

Here, and in the text below, $\nu_{\alpha n} = \sigma_n n_n v_{T\alpha}$, where $v_{T\alpha}^2 = \kappa T_\alpha/m_\alpha$ and σ_n is the cross section of the neutrals. The parallel electron motion is described by

$$0 = \frac{e}{m_e}\frac{\partial\phi}{\partial z} - \frac{v_{Te}^2}{n_e}\frac{\partial n_e}{\partial z} - \nu_{en}v_{ez}. \quad (11)$$

The perpendicular motion introduces the following terms in the electron continuity equation:

$$\nabla_\perp\cdot(n_e\mathbf{v}_{\perp e}) = \nabla_\perp\left(\frac{n_e}{B_0}\boldsymbol{e}_z \times \nabla_\perp\phi - \frac{v_{T_e}^2}{\Omega_e}\boldsymbol{e}_z \times \nabla_\perp n_e - \frac{v_{T_e}^2}{\Omega_e}\frac{\nu_e}{\Omega_e}\nabla_\perp n_e + \frac{\nu_e}{\Omega_e}\frac{\nabla_\perp\phi}{B_0}\right).$$

The non-zero terms, being multiplied by ν_e/Ω_e and due to (5), can be neglected. As a result, the electron continuity equation yields

$$\frac{n_{e1}}{n_{e0}} = \frac{iek_z^2\phi_1}{m_e\nu_{en}(\omega - \mathbf{k}\cdot\mathbf{v}_{e0}) + ik_z^2\kappa T_e}. \quad (12)$$

In the case of an arbitrary ratio of the ion-neutral collision frequency and the wave frequency, the fluid equations for ions may become unable of capturing the essential physical effects and the kinetic description may be needed. Hence, the ion dynamics is described by the Boltzman kinetic equation with the Krook's collisional term, which is good enough for ion collisions with neutral having nearly equal mass

$$\frac{\partial f_1}{\partial t} + \mathbf{v}\frac{\partial f_1}{\partial \mathbf{r}} + \frac{e}{m_i}(\mathbf{E}_1 + \mathbf{v}\times\mathbf{B}_0)\frac{\partial f_1}{\partial \mathbf{v}} = -\nu_{in} f_1.$$

This yields the perturbed distribution function described by

$$f_1 = -\frac{ie\nabla\phi_1\cdot\mathbf{v}}{m_i v_{Ti}^2(\omega + i\nu_{in} - \mathbf{k}\cdot\mathbf{v})} f_0. \tag{13}$$

Performing the usual integration for the ions we obtain

$$\frac{n_{i1}}{n_{i0}} = -\frac{e\phi_1}{m_i v_{Ti}^2}\left[1 - J_+\left(\frac{\omega_i}{k v_{Ti}}\right)\right]. \tag{14}$$

Here, $J_+(\alpha) = [\alpha/(2\pi)^{1/2}]\int_c d\zeta \exp(-\zeta^2/2)/(\alpha-\zeta)$ is the plasma dispersion function, $\zeta = v/v_{Ti}$, and $\omega_i = \omega + i\nu_{in}$. Using the quasi-neutrality we obtain the dispersion equation:

$$\frac{1}{\kappa T_i}\left[1 - J_+\left(\frac{\omega_i}{k v_{Ti}}\right)\right] + \frac{ik_z^2}{m_e\nu_{en}(\omega - \mathbf{k}\cdot\mathbf{v}_{e0}) + ik_z^2\kappa T_e} = 0. \tag{15}$$

Expanding $J_+(\alpha)$ in the limit $|\alpha| \gg 1$ and $|\mathcal{R}e(\alpha)| \gg |\mathcal{I}m(\alpha)|$ we obtain the dispersion equation

$$\begin{aligned}(\omega + i\nu_{in})^2 &= k^2c_s^2 + \frac{3k^4v_{Ti}^2c_s^2}{\omega^2}\\ &- 6\nu_{en}\nu_{in}\frac{m_e}{m_i}\frac{k^2}{k_z^2}\frac{k^2v_{Ti}^2}{\omega^3}(\omega - \mathbf{k}\cdot\mathbf{v}_{e0}) - i6k^4v_{Ti}^2c_s^2\frac{\nu_{in}}{\omega^3}\\ &- i\nu_{en}\frac{m_e}{m_i}\frac{k^2}{k_z^2}\left(1 + \frac{3k^2v_{Ti}^2}{\omega^2}\right)(\omega - \mathbf{k}\cdot\mathbf{v}_{e0})\\ &- i\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{\omega^3}{kv_{Ti}} + i\frac{3\omega^2\nu_{in}}{kv_{Ti}}\right)\left[\tau - i\frac{m_e\nu_{en}(\omega - \mathbf{k}\cdot\mathbf{v}_{e0})}{m_ik_z^2v_{Ti}^2}\right].\end{aligned} \tag{16}$$

This, under the condition that $\omega = \omega_r + i\omega_{im}$, $|\omega_{im}| \ll |\omega_r|$ yields the spectrum:

$$\omega_r^2 \approx k^2c_s^2\left(1 + \frac{3}{\tau}\right) - 6\frac{\nu_{en}\nu_{in}}{\tau}\frac{m_e}{m_i}\frac{k^2}{k_z^2}\left(1 - \frac{\mathbf{k}\cdot\mathbf{v}_{e0}}{kc_s}\right). \tag{17}$$

In the imaginary part we keep $\omega_r^2 \approx k^2c_s^2(1 + 3/\tau)$ and obtain the increment/decrement approximately:

$$\begin{aligned}\omega_{im} &\approx -\nu_{in}\left[1 + \frac{3}{\tau(1+3/\tau)^2}\right]\\ &- \nu_{en}\frac{m_e}{2m_i}\frac{k^2}{k_z^2}\left[1 + \frac{3}{\tau(1+3/\tau)}\right]\left[1 - \frac{\mathbf{k}\cdot\mathbf{v}_{e0}}{kc_s(1+3/\tau)^{1/2}}\right]\\ &- kc_s\left(\frac{\pi}{8}\right)^{1/2}\tau^{3/2}\left(1 + \frac{3}{\tau}\right)\exp\left[-\frac{\tau}{2}(1+3/\tau)\right].\end{aligned} \tag{18}$$

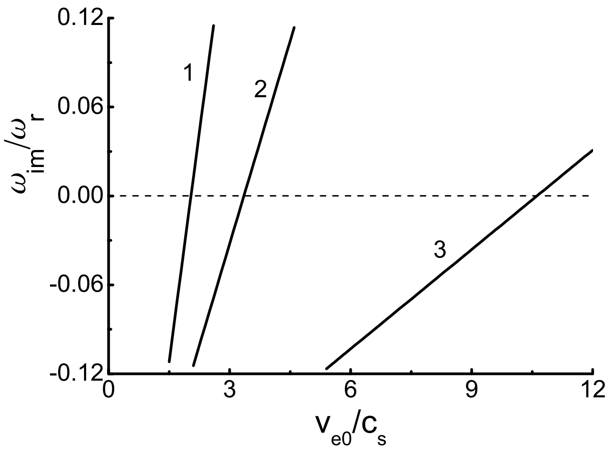

Fig. 1. The imaginary part of the frequency (in units of ω_r) in terms of the electron drift velocity for three values of the angle $\psi = 7.5°$ (line 1), $\psi = 11.25°$ (line 2), and $\psi = 22.5°$ (line 3).

It is seen that there can be no instability in the case of a counter propagating wave with respect to the drift velocity, i.e. if $\mathbf{k}\cdot\mathbf{v}_{e0} < 0$. In the opposite case, the instability sets in if

$$\begin{aligned}\frac{v_{e0}}{c_s} > \frac{(1+3/\tau)^{1/2}}{\cos\psi}&\left\{1 + \left[\nu_{in}\left(1 + \frac{3}{\tau(1+3/\tau)^2}\right)\right.\right.\\ &\left.+kc_s\left(\frac{\pi}{8}\right)^{1/2}\tau^{3/2}\left(1+\frac{3}{\tau}\right)\exp\left[-\frac{\tau}{2}\left(1+\frac{3}{\tau}\right)\right]\right]\\ &\left.\times\frac{2m_ik_z^2}{m_e\nu_{en}k^2[1+3/(\tau+3)]}\right\}.\end{aligned} \tag{19}$$

Here, ψ is the angle between $\mathbf{k}$ and $\mathbf{v}_{e0}$, and the factor $3/\tau$ can be omitted for larger values of τ. This situation is presented in Figure 1, where the ratio ω_{im}/ω_r is plotted in terms of the normalized electron drift for the normalized value $\nu_{en}/\omega_r = 10$ and consequently for $\nu_{in}/\omega_r = (\nu_{en}/\omega_r)[T_im_e/(T_em_i)]^{1/2}$, where $\tau = 10$, and the angle ψ takes values 7.5°, 11.25°, 22.5° for the lines 1–3, respectively. The fast growing mode, driven by the electron current, has a threshold which shifts towards larger values of v_{e0}/c_s, for larger values of the angle ψ. For the lines 1–3 the mode becomes unstable for v_{e0} exceeding $2c_s$, $3.3c_s$ and $10.7c_s$, respectively. Note that these values are well below (1).

The shift of the instability threshold in the limit $\nu_{in} \ll \omega_r$ and for $\tau = 10$ is presented in Figure 2, for several values of the electron collision frequency and for the angle ψ in the range $0 < \psi < \pi/2$. From (19) one finds that in this limit the instability condition becomes

$$\frac{v_{e0}}{c_s} > \frac{1}{\cos\psi}\left(1 + 377\frac{kc_s}{\nu_{en}}\sin^2\psi\right). \tag{20}$$

Here, the limit $\psi \to 0$, which corresponds to wave propagating along the electron drift, is not justified as it violates the conditions introduced for the electron dynamics. For an electron-proton plasma and for the ion sound, this approximately means that there must be a $k_z > k/43$, or

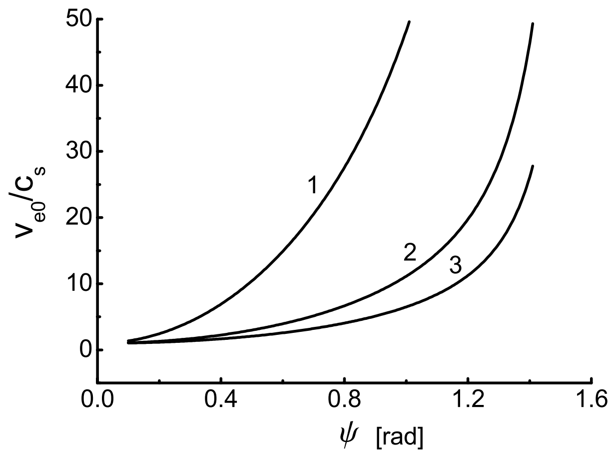

Fig. 2. Threshold value of the perpendicular electron drift in terms of the angle between the drift velocity vector and the wave-vector. The lines 1−3 are for the ratio kc_s/ν_{en} 1/10, 1/50, 1/100, respectively. Unstable values of the drift are above the lines.

$\psi > 1.33°$. Similarly, the limit $\psi \to \pi/2$ implies propagation perpendicular to the drift and an infinite drift magnitude for the instability. It is seen that, depending on the angle of the propagation, the threshold can be much smaller compared to (1), which according to (19) appears, firstly, due to the oblique propagation because the critical term is multiplied by the square of k_z/k (i.e., by $\sin^2\psi$), and, secondly, due to the collisions (i.e., the term $kc_s/\nu_{en} \ll 1$).

4 An additional hotter electron component

Next, we consider the case when an additional population of electrons with a higher temperature is present in the considered plasma. Studies on such plasmas are available in the literature, both in laboratory [11] and space plasma context [12]. In the photosphere and in the terrestrial ionosphere such a situation may occur due to an inflow of hotter electrons from upper layers, e.g., due to reconnection processes in which a potential difference is created [13] resulting in an acceleration of the electrons along the magnetic field lines. Observations in the auroral regions [12] reveal the existence of a dominant hotter electron population with energies measured in keV, and a cold population with an energy of a few tens of eV. In the weakly ionized plasma discussed here, the thermalization of the two electron groups is slow because collisions with heavy neutrals (which, due to the huge difference in mass, is inefficient to considerably change the electron energy) is dominant, so that such a two-temperature electron plasma can last relatively long. The quasi-static equilibrium equations (2), (3) are supplemented by one more equation

$$-en_{h0}(\mathbf{E}_0 + \mathbf{v}_{h0} \times \mathbf{B}_0) - m_e n_{h0}\nu_{hn}(\mathbf{v}_{h0} - \mathbf{v}_{n0}) = 0. \quad (21)$$

Here, the index h denotes the hotter electron component. The quasi-neutrality condition now reads $n_{i0} = n_{e0} + n_{h0}$, and we also allow for the hotter electron component to have an additional velocity along the magnetic field lines which may be determined by external conditions as mentioned above.

In the quasi-static stage, the ion flux becomes equal to the sum of the two electron fluxes:

$$n_{i0}v_{i0x} = n_{e0}v_{e0x} + n_{h0}v_{h0x}, \quad (22)$$

which yields approximatively the electric field

$$E_0 = \frac{v_{n0}}{e}\frac{n_{e0}\nu_{en}/a_2 + n_{h0}\nu_{hn}/a_3 - n_{i0}\nu_{in}/a_1}{n_{i0}/(a_1 m_i) + n_{e0}/(a_2 m_e) + n_{h0}/(a_3 m_e)}, \quad (23)$$

where

$$a_1 = \nu_{in} + \frac{\Omega_i^2}{\nu_{in}}, \quad a_2 = \nu_{en} + \frac{\Omega_e^2}{\nu_{en}}, \quad a_3 = \nu_{hn} + \frac{\Omega_e^2}{\nu_{hn}}.$$

The contribution of the h electron component to the induced electric field depends on its temperature and concentration. For a high temperature it may be that

$$\Omega_e \ll \nu_{hn}. \quad (24)$$

In this case, the h electrons behave as un-magnetized and the electric field is reduced. Also, in this limit, their drift in the y-direction is negligible similar to that of the ions. Using conditions (5) and (24) the electric field (23) becomes

$$E_0 = -\frac{v_{n0}}{e}\left(\nu_{in}\Omega_e^2/\nu_{en}\right) / \left[\nu_{in}/m_e + n_{i0}\Omega_e^2/\left(n_{e0}m_i\nu_{en}\right) + \left(n_{i0}/n_{e0} - 1\right)\nu_{in}\Omega_e^2/\left(m_e\nu_{hn}\nu_{en}\right)\right]. \quad (25)$$

Obviously, because of the larger denominator, this yields a smaller electric field than equation (6). This is expected because of the increased collisions and, consequently, more effective drag by neutrals. On the other hand, a higher relative concentration of the electron population h results in a reduced electric field, and vice versa. This is also related to higher collision rates and, consequently, lower magnetization, yielding a smaller induced electric field, and vice versa. It can be easily shown that for a colder h population the effect is negligible.

5 Effects of a hotter electron specie on the ion sound

To study the effect of the h population on the ion sound, we use the same conditions as before. The parallel and perpendicular momentum equations together with the continuity equation of the h component yield an additional equation:

$$\frac{n_{h1}}{n_{h0}} = \frac{iek_z^2\eta\phi_1}{m_e\nu_{hn}(\omega - \mathbf{k}_\perp \cdot \mathbf{v}_{h0} - k_z v_{h0z}) + i\eta k_z^2\kappa T_h}, \quad (26)$$

$$\eta = 1 + \frac{k_\perp^2}{k_z^2}\frac{\nu_{hn}^2}{\Omega_e^2}\frac{1}{1 + \nu_{hn}^2/\Omega_e^2}.$$

Here, contrary to (12), $\eta \neq 1$ because of the arbitrary ratio ν_{hn}/Ω_e. Equation (26) is combined with equations (12), (14), with the assumption of quasi-neutrality, yielding

$$\frac{1}{\kappa T_i}\left(1+\frac{n_{h0}}{n_{e0}}\right)\left[1-J_+\left(\frac{\omega_i}{kv_{Ti}}\right)\right] + \frac{ik_z^2}{m_e\nu_{en}(\omega-\mathbf{k}_\perp\cdot\mathbf{v}_{e0})+ik_z^2\kappa T_e} + \frac{i\eta k_z^2 n_{h0}/n_{e0}}{m_e\nu_{hn}(\omega-\mathbf{k}_\perp\cdot\mathbf{v}_{h0}-k_z v_{h0z})+i\eta k_z^2\kappa T_h}=0, \quad (27)$$

where $\omega_i=\omega+i\nu_{in}$. Neglecting all dissipative effects yields the modified dispersion equation for the Landau damped ion sound:

$$\omega_r^2=k^2c_s^2\left(\gamma+\frac{3}{\tau}\right), \quad \gamma=\frac{T_h}{T_e}\,\frac{(n_{h0}/n_{e0}+1)}{n_{h0}/n_{e0}+T_h/T_e}, \quad (28)$$

$$\omega_{im}\approx-\omega_r\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{\gamma T_e}{T_i}\right)^{1/3}\exp\left[-\left(\frac{\gamma T_e}{2T_i}\right)\right]. \quad (29)$$

Comparing this with the standard e-i collision case [6], it is seen that the hotter electron specie makes the electron-ion temperature ratio effectively larger. Here, $\gamma>1$ as long as $T_h/T_e>1$, so the frequency is higher. Note also that $\gamma\to T_h/T_e$ when $n_{h0}/n_{e0}\gg 1$ and $n_{h0}/n_{e0}\gg T_h/T_e$, and $\gamma\to 1$ when $n_{h0}/n_{e0}\to 0$. Hence, in the case of the solar photosphere or the terrestrial ionosphere the inflow of hotter electrons implies the possibility of an ion sound wave with a frequency that can far exceed the frequency which is normally expected from the local plasma parameters. Yet, whether such a wave will indeed be excited or not, may be deduced only by solving the full dispersion equation (27). Omitting the ion collision terms this dispersion relation reduces to

$$\omega^2\approx\frac{\delta_1}{\mathcal{D}}\left(1+\frac{3k^2v_{Ti}^2}{\omega^2}\right)\left[k^2c_s^2(1+\alpha_1\eta\delta)-\frac{k^2}{k_z^2}\frac{m_e}{m_i}\nu_{en}\omega_1\beta_1\eta\delta\right] - i\,\frac{\delta_1}{\mathcal{D}}\left[\delta_1\nu_{en}\omega_1\frac{k^2}{k_z^2}\frac{m_e}{m_i}(1+\alpha_1\eta\delta)+\beta_1\eta\delta k^2c_s^2\right] - i\tau\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega^3}{kv_{Ti}}\frac{\delta_1}{\mathcal{D}}\left[1+\alpha_1\eta\delta-\frac{\nu_{en}\omega_1\beta_1\eta\delta}{k_z^2v_{Te}^2}\right]\times\exp\left(-\frac{\omega^2}{2k^2v_{Ti}^2}\right). \quad (30)$$

Here,

$$\omega_1=\omega-\mathbf{k}_\perp\cdot\mathbf{v}_{e0}, \quad \omega_2=\omega-\mathbf{k}_\perp\cdot\mathbf{v}_{h0}-k_zv_{h0z},$$

$$\delta=\frac{n_{h0}}{n_{e0}}, \quad \delta_1=1+\frac{n_{h0}}{n_{e0}}, \quad \mathcal{D}=(1+\alpha_1\eta\delta)^2+(\beta_1\eta\delta)^2,$$

$$\alpha_1=\frac{\dfrac{\nu_{en}\nu_{hn}\omega_1\omega_2}{k_z^4v_{Th}^4}+\dfrac{T_e\eta}{T_h}}{\eta^2+\dfrac{\nu_{hn}\omega_2^2}{k_z^4v_{Th}^4}}, \quad \beta_1=\frac{\dfrac{\nu_{hn}\omega_2}{k_z^2v_{Th}^2}\dfrac{T_e}{T_h}-\dfrac{\eta\nu_{en}\omega_1}{k_z^2v_{Th}^2}}{\eta^2+\dfrac{\nu_{hn}\omega_2^2}{k_z^4v_{Th}^4}}.$$

Equation (30) can be drastically simplified in the limit when ν_{hn}/k_zv_{Th} is not much larger than 1. This together with $\omega_2/k_zv_{Th}\ll 1$ yields $\alpha_1=T_e/(\eta T_h)$, and β_1 becomes negligible. The drift/flow effects and the perpendicular dynamics of the h component in this limit both vanish, and the spectrum is approximately given by (28), while the imaginary part of the frequency becomes

$$\omega_{im}=-\nu_{en}\frac{\gamma}{2}\frac{k^2}{k_z^2}\frac{m_e}{m_i}\left(1-\frac{\mathbf{k}\cdot\mathbf{v}_{e0}}{\omega_r}\right) - \omega_r\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{\gamma T_e}{T_i}\right)^{1/3}\exp\left[-\left(\frac{\gamma T_e}{2T_i}\right)\right]. \quad (31)$$

Here, using ω_r in which the τ term is omitted, one finds that the instability sets in under the condition

$$\frac{v_{e0}}{c_s}>\frac{1}{\cos\psi}\left[\gamma^{1/2}+\frac{m_ic_sk_z^2}{m_e\nu_{en}k}\left(\frac{\pi}{2}\right)^{1/2}(\gamma\tau)^{3/2}\exp\left(-\frac{\gamma\tau}{2}\right)\right]. \quad (32)$$

Since $\gamma>1$ it is clear that, compared to (19), the instability threshold is higher.

6 Conclusions

We have discussed the effects of the motion of neutrals on the behavior of the ion sound mode in a weakly ionized plasma. Plasmas of that kind are frequently encountered both in the laboratory [1,3], and in space [5,7]. Collisions of plasma species with neutrals in such plasmas play a decisive role and involve possibly different sets of (kinetic) equations for ions, and (fluid) equations for electrons. For the same reasons, electrons may behave as magnetized while ions may remain un-magnetized and, consequently, the dynamics of the two species becomes very different [10]. In the case of an ion sound propagating in such a system, there is no preferential direction for the ion motion, while the electron dynamics is strongly influenced by the magnetic field. As a result, the spectrum and the increment/decrement of the mode becomes different compared to the standard ion-electron case. This is particularly obvious for the mode increment, which in a plasma with neutrals may increase and, in the same time, the instability threshold may decrease. In fact, the instability itself is fed by the motion of neutrals, which is transferred to the plasma species making the mode unstable.

An additional specie of hotter electrons, which is frequently observed in the auroral regions of the terrestrial ionosphere, introduces certain changes in the mode behavior. These hotter electrons are more collisional and, therefore, they perform a motion similar to that of the ions. As a result, the induced electric field becomes smaller due to the fact that a part of the ion charge is shielded by the hotter electron specie moving nearly in the same manner, while in the same time the instability threshold is changed and increased. We underline an essential difference between the standard electron-drift driven instability condition equation (1) and our current results equations (18) and (31). It is seen that, neglecting the τ contribution, the necessary instability condition in the standard case, $v_{e0} > c_s$, is more easily satisfied compared to the present case where $v_{e0} > c_s/\cos(\psi)$. However, the actual sufficient instability condition is, in fact, much more easily satisfied in the present case.

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